Quantum Violations of the Equivalence Principle in a Modified Schwarzschild Geometry. Neutrino Oscillations

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December 21, 2000

Abstract

Neutrino flavour oscillations are analyzed in a model in which particles experience an effective Schwarzschild geometry modified by maximal acceleration corrections. These imply a quantum violation of the equivalence principle. The corresponding shifts in the phase of the neutrino mass eigenstates are calculated and discussed.

PACS: 04.70.-s, 04.70.Bw
Keywords: Neutrino Oscillations, Quantum Geometry, Maximal Acceleration

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Einstein's equivalence principle plays a fundamental role in the construction and testing of theories of gravity. Though verified experimentally to better than a part in 10^{11} for bodies of macroscopic dimensions [1, 2], doubts have at times been expressed as to its validity down to microscopic scales. It is conceivable, for instance, that the equality of inertial and gravitational mass break down for antimatter [3, 4], or in quantum field theory at finite temperatures [5]. Quantum violations of the equivalence principle have also been discussed in Ref. [6]

Einstein's equivalence principle is also violated in a model developed by Caianiello and collaborators [7]–[9] as a first step toward the unification of quantum mechanics and general relativity. The model interprets quantization as curvature of the eight-dimensional space-time tangent bundle TM. In this space the standard operators of the Heisenberg algebra are represented as covariant derivatives and the quantum commutation relations are interpreted as components of the curvature tensor.

The merits of the model are extolled by its intrinsic simplicity and the connections it establishes among seemingly unrelated research areas. For instance, the model incorporates the notion that the proper acceleration of massive particles has an upper limit $\mathcal{A}_m$. Classical and quantum arguments supporting the existence of a maximal acceleration (MA) have long been given [10]–[19]. MA also appears in the context of Weyl space [20]–[23] and of a geometrical analogue of Vigier's stochastic theory [24].

$\mathcal{A}_m$ is regarded by some as a universal constant fixed by Planck's mass [25]–[28], but a direct application of Heisenberg's uncertainty relations [29, 30] and the geometrical interpretation of the quantum commutation relations given by Caianiello, suggest that $\mathcal{A}_m$ be fixed by the rest mass of the particle itself according to $\mathcal{A}_m = 2mc^3/\hbar$. It is precisely through $\mathcal{A}_m$ that the equivalence principle is violated.

MA delves into a number of questions. The existence of a MA would rid black hole entropy of ultraviolet divergencies [31]–[33], and circumvent inconsistencies associated with the application of the point-like concept to relativistic quantum particles [34, 35].

A limit on the acceleration also occurs in string theory. Here the upper limit manifests itself through Jeans-like instabilities [36, 37] which occur when the acceleration induced by the background gravitational field is larger than a critical value $a_c = (m\alpha)^{-1}$for which the string extremities become causally disconnected [38, 39]. m is the string mass and α is the string tension. Frolov and Sanchez [40] have then found that a universal critical acceleration a_c must be a general property of strings. It is moreover possible to derive from it the generalized uncertainty principle of string theory [41].

Applications of Caianiello's model include cosmology [27, 28], where the initial singularity can be avoided while preserving inflation, the dynamics of accelerated strings [42], the energy spectrum of a uniformly accelerated particle [43], the periodic structure as a function of momentum in neutrino oscillations [43] and the expansion of the very early universe [38, 39].

The extremely large value that $\mathcal{A}_m$ takes for all known particles makes a direct test of the model difficult. Nonetheless a direct test that uses photons in a cavity has also been suggested [44]. More recently, we have worked out the consequences of the model for the classical electrodynamics of a particle [45], the mass of the Higgs boson [46, 47] and the Lamb shift in hydrogenic atoms [48]. In the last instance, the agreement between experimental data and MA corrections is very good for H and D. For He^+ the agreement between theory and experiment is improved by 50% when MA corrections are included. MA effects in muonic atoms appear to be measurable in planned experiments [49]. MA also affects the helicity and chirality of particles [50].

Very recently we have studied the behaviour of classical [51] and quantum [52] particles in a Schwarzschild field with MA modifications. In all these works space-time is endowed with a causal structure in which the proper accelerations of massive particles are limited. This is achieved by means of an embedding procedure pioneered in [43] and further discussed in [51]. The procedure stipulates that the line element experienced by an accelerating particle is represented by

$$d\tau^2 = \left(1 + \frac{g_{\mu\nu}\ddot{x}^\mu\ddot{x}^\nu}{\mathcal{A}_m^2}\right) g_{\alpha\beta}dx^\alpha dx^\beta \equiv \sigma^2(x) g_{\alpha\beta}dx^\alpha dx^\beta \,, \tag{1}$$

and is therefore observer-dependent as conjectured by Gibbons and Hawking [53]. As a consequence, the effective space-time geometry experienced by accelerated particles exhibits mass-dependent corrections, which in general induce curvature, and give rise to a mass-dependent violation of the equivalence principle. The classical limit $(\mathcal{A}_m)^{-1} = \frac{\hbar}{2mc^3} \to 0$ returns space-time to its ordinary geometry.

In Eq. (1) $\ddot{x}^\mu = d^2x^\mu/ds^2$ is the, in general, non–covariant acceleration of a particle along its worldline. Caianiello's effective theory is therefore intrinsically non-covariant. Nonetheless the choice of $\ddot{x}^\mu$ is supported by the derivation of $\mathcal{A}_m$ from quantum mechanics, by special relativity and by the weak field approximation to general relativity.

The embedding procedure also requires that $\sigma^2(x)$ be present in (1) and that it be calculated in the same coordinates of the unperturbed gravitational background. The model is not intended, therefore, to supersede general relativity, but rather to provide a way to calculate the quantum corrections to the structure of space-time implied by Eq.(1).

In this work we ask ourselves whether the violations of Einstein's equivalence principle mentioned above have observable consequences.

The best opportunity to observe an effect of this kind is perhaps in connection with neutrino oscillations, as pointed out by Gasperini [54] and Halprin and Leung [55].

Neutrino oscillations can occur in vacuum if the eigenvalues of the mass matrix are not all degenerate and the corresponding mass eigenstates differ from the weak eigenstates. Flavour oscillations have frequently been advocated as possible explanations of the solar neutrino deficiency and of the atmospheric neutrino problem. The most discussed version of this type of solutions is the MSW effect [56, 57] in which the oscillations are enhanced by matter in the Sun's interior.

Gravitational fields per se can not induce neutrino oscillations because gravity couples universally to all kinds of matter. Neutrino oscillations can however be induced by violations of the equivalence principle. In this case oscillations arise if the coupling of neutrinos to gravity is non-diagonal relative to neutrino flavours. Only two–neutrino oscillations will be considered in this work. A related calculation [43] was performed for the two–dimensional problem of particles in hyperbolic motion in a Kruskal plane. These particles are static relative to Schwarzschild coordinates. This restriction is removed below.

For convenience, the natural units $\hbar = c = G = 1$ are used below. The conformal factor can be easily calculated as in [51] starting from (2), with $\theta = \pi/2$, and from the well known expressions for $\ddot{t}, \ddot{r}$ and $\ddot{\phi}$ in Schwarzschild coordinates [58]. One obtains

$$\sigma^2(r) = 1 + \frac{1}{\mathcal{A}_m^2}\left\{-\frac{1}{1-2M/r}\left(-\frac{3M\tilde{L}^2}{r^4} + \frac{\tilde{L}^2}{r^3} - \frac{M}{r^2}\right)^2 + \right.$$
$$\left. + \left(-\frac{4\tilde{L}^2}{r^4} + \frac{4\tilde{E}^2 M^2}{r^4(1-2M/r)^3}\right)\left[\tilde{E}^2 - \left(1 - \frac{2M}{r}\right)\left(1 + \frac{\tilde{L}^2}{r^2}\right)\right]\right\}, \tag{2}$$

where M is the mass of the source, $\tilde{E}$ and $\tilde{L}$ are the total energy and angular momentum per unit of test particle rest mass m. In the weak field approximation, the modifications to the Schwarzschild geometry experienced by *radially* accelerating neutrinos follows from (2). One gets

$$\sigma^2(r) = 1 - \frac{1}{\mathcal{A}_m^2}\left(\frac{1}{4} + \frac{E^2}{m^2} - \frac{E^4}{m^4}\right)\frac{r_s^2}{r^4}, \tag{3}$$

where $r_s = 2M$ is the Schwarzschild radius and E is the total energy. We neglect spin contributions. The effective Hamiltonian for two–neutrino oscillations can be derived from the Klein–Gordon equation of Ref. [52]. Ignoring terms proportional to the identity matrix and derivatives of σ, one obtains

$$H \sim \sqrt{E^2 - m^2\sigma^2} \sim E - \frac{m^2\sigma^2}{2E}, \tag{4}$$

in the approximation $m^2\sigma^2/2E^2 < 1$. The additional corrections to the conventional two–neutrino oscillations are therefore given by

$$\frac{m^2}{2E\mathcal{A}_m^2}\left(\frac{1}{4} + \frac{E^2}{m^2} - \frac{E^4}{m^4}\right)\frac{r_s^2}{r^4}. \tag{5}$$

The first term in (5) is independent of m and will be dropped. The second term is compatible with the approximation for $r > (r_s^2/4m^2)^{1/4} \equiv r_{c1}$ which gives $r > 2 \times 10^{-2}$m if r_s refers to the Sun and $m \sim 0.1$eV. If r_s is that of Earth, the condition becomes $r > 2 \times 10^{-5}$m. The last term in (5) satisfies the compatibility condition for $r > (E^2 r_s^2/4m^4)^{1/4} \equiv r_{c2}$ or $r > 3 \times 10^3$m for $m \sim 0.1$eV, $r_s \sim 10^3$m (Sun) and $E \sim 1$GeV. Lower values of E and r_s make the condition easier to meet. One also finds $r_{c2} = (E/m)^{1/2} r_{c1}$.

Following Refs. [54, 55] and taking into account *medium effects*, one finds

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}=\frac{1}{2}\begin{pmatrix}2\sqrt{2}\,G_F N_e(r)-\dfrac{\tilde\Delta}{r^4}\cos 2\theta & \dfrac{\tilde\Delta}{r^4}\sin 2\theta\\ \dfrac{\tilde\Delta}{r^4}\sin 2\theta & 0\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}, \qquad (6)$$

where

$$\tilde\Delta=\frac{\Delta m^2 r^4}{2E}\left(1-\frac{r_s}{r}\right)+\frac{r_s^2E^3\Delta m^2(m_1^2+m_2^2)}{8m_1^4m_2^4}\left[1-\frac{m_1^2m_2^2}{E^2(m_1^2+m_2^2)}\right] \qquad (7)$$

and $\Delta m^2=m_2^2-m_1^2$. The second and forth terms in (7) are just corrections of the other two and will be dropped for simplicity. For ultra-relativistic neutrinos one has $r\sim t$, and the equation of evolution can be re–cast in the form

$$i\dot\nu_e = \left[\sqrt{2}\,G_F N_e(t)-\frac{\tilde\Delta\cos 2\theta}{2t^4}\right]\nu_e+\frac{\tilde\Delta}{2t^4}\sin 2\theta\nu_\mu \qquad (8)$$

$$i\dot\nu_\mu = \frac{\tilde\Delta\sin 2\theta}{2t^4}\nu_e\,. \qquad (9)$$

Eqs. (8) and (9) can be de–coupled and the equation of evolution of the flavour eigenstate ν_μ is

$$\ddot\nu_\mu+\left[\frac{4}{t}+\frac{i}{2}\left(2\sqrt{2}G_FN_e(t)-\frac{\tilde\Delta\cos 2\theta}{t^4}\right)\right]\dot\nu_\mu+\left(\frac{\tilde\Delta\sin 2\theta}{2t^4}\right)^2\nu_\mu=0\,. \qquad (10)$$

¿From Eq. (6) one derives the resonance condition

$$\cos 2\theta=\frac{2\sqrt{2}\,G_FN_e(r)r^4}{\tilde\Delta}, \qquad (11)$$

where, for the Sun, $N_e(r)\sim N_0\exp(-10.54r/R_\odot)\mathrm{cm}^{-3}$, $N_0=85N_A\mathrm{cm}^{-3}$. N_A is Avogadro's number. In the case of the Sun $\sqrt{2}G_fN_e\sim 10^{-12}$eV, while for a supernova $\sqrt{2}G_FN_e\sim 1$eV. Eq. (11) is therefore satisfied by $\cos 2\theta\approx 0$ even for high values of r. At resonance, Eq. (10) reduces to the form

$$\ddot\nu_\mu+\frac{4}{t}\dot\nu_\mu+\left(\frac{\tilde\Delta}{2t^4}\right)^2\nu_\mu=0\,. \qquad (12)$$

Taking $m_2\sim m_1\sim m$, one finds

$$\tilde\Delta\approx\frac{\Delta m^2r^4}{2E}+\frac{E^3r_s^2}{4}\frac{\Delta m^2}{m^6}\equiv\Delta\Phi_{(0)}+\Delta\Phi_{\mathcal{A}_m}\,. \qquad (13)$$

The drastically different behaviours in E and r of the two terms in (13) now require some discussion. The phase generated by the MA corrections is, in particular, proportional to r^{-3} which indicates its potential relevance at short distances from the neutrinos source. The two terms become comparable in size at a distance $r_0 = E(r_s/\sqrt{2}m^3)^{1/2} = (E\sqrt{2}/m)^{1/2} r_{c2}$. For $r_{c2} < r < r_0$ the MA correction term predominates, but subsides rapidly in the region $r > r_0 > r_{c2}$ where the conventional term takes over. There are therefore two possibilities to consider.

i) $r_{c2} < r < r_0$. In this case the solution is

$$\nu_\mu = a e^{i\tilde{\Delta}/6t^3} + b e^{-i\tilde{\Delta}/6t^3}\,, \tag{14}$$

where a and b are constants and $\tilde{\Delta} \sim \Delta\Phi_{\mathcal{A}_m}$. The oscillatory behaviour predominates when $\tilde{\Delta}/12r^3 \sim 1$, which gives the characteristic length $L_0 \sim (\tilde{\Delta}/12)^{1/3}$. For $r > L_0$, the oscillations decrease rapidly. There is therefore a sphere of radius

$$L_0 \sim \frac{E}{2m^2}\left(\frac{\Delta m^2 r_s^2}{6}\right)^{1/3} > r_s$$

within which neutrino oscillations take place at a significant rate. This sphere is external to the impenetrable shell discussed in [51] and [52]. The ratio

$$\frac{L_0}{r_0} = \frac{(\Delta m^2)^{1/3} r_s^{1/6}}{3.1 m^{1/2}}$$

becomes unity for $\Delta m^2 \sim 28.5\sqrt{m^3/r_s}$. If $\Delta m^2 > 28.5\sqrt{m^3/r_s}$, then $r_0 < L_0$ and this oscillation mechanism becomes less significant. This may be illustrated numerically as follows. In the case of the Sun, $r_s \sim 10^3$m and for $m \sim 0.1$eV, $\Delta m^2 \sim 10^{-2}\text{eV}^2$, $E \sim 10$MeV, one finds $r_0 \sim 6.2 \times 10^5$m and $L_0 \sim 6.5 \times 10^7$m, which indicates that a negligible fraction of ν_e would be converted into ν_μ by the mechanism discussed.

For atmospheric neutrinos in the gravitational field of Earth ($r_s \sim 8 \times 10^{-3}$m) and the values $E \sim 1$GeV, $m \sim 0.1$eV, $\Delta m^2 \sim 10^{-2}\text{eV}^2$ one obtains $r_0 \sim L_0 \sim 10^6$m which indicates appreciable conversion in regions of space surrounding Earth.

ii) $r > r_0 > r_{c2}$. For these values of r the MA corrections become negligible, $\tilde{\Delta} \sim \Delta\Phi_{(0)}$ and Eq. (12) becomes

$$\ddot{\nu}_\mu + \frac{4}{t}\dot{\nu}_\mu + \omega^2 \nu_\mu = 0\,, \tag{15}$$

where $\omega = \Delta m^2/4E$. The solution of (15) is

$$\nu_\mu = \frac{f}{(\omega t)^{3/2}} Z_{-3/2}(\omega t) = \frac{f}{(\omega t)^2}\left(\sin\omega t + \frac{1}{\omega t}\,\cos\omega t\right)\,, \tag{16}$$

where f is a constant. The amplitude of the oscillations is damped for $r > 4E/\Delta m^2$.

In summary, the quantum violations of the equivalence principle predicted by Caianiello's model lead to neutrino oscillations that are characterized, at resonance, by two lengths, r_0 and L_0. For $r < r_0$ the oscillations induced by MA dominate. L_0 is the value of r for which the induced phase gives a relevant contribution. Ideally, one would have $r_0 \sim L_0$. This condition is satisfied if $\Delta m^2 \simeq 28.5\sqrt{m^3/r_s}$ which favours larger neutrino masses and situations in which the gravitational source has a small r_s. It is the case, for instance, of atmospheric neutrinos in the gravitational field of the Earth.

For $r > r_0$ the importance of the equivalence principle violations induced by MA decreases rapidly and the conventional (damped) vacuum oscillations dominate. Finally, the present calculations underscore the basic point of Gasperini, Halprin and Leung that violations of the equivalence principle are important in neutrino oscillations. At the same time, the calculations also show that this statement is not meant to apply universally and that a detailed model of the violations may provide different results for different gravitational sources, physical situations, regions of space and values of the parameters involved.

Acknowledgements

Research supported by MURTS fund PRIN 99 and the Natural Sciences and Engineering Research Council of Canada. G.L. aknowledges the financial support of UE (P.O.M. 1994/1999).

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